QUIZ 6 SOLUTIONS: LESSON 7
SEPTEMBER 12, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [10 pts] When an object is removed from a furnace and placed in a room with a constant temperature of 75°F, its temperature is 1000°F. One hour after it is removed, the temperature of the object is 500°F. Find the temperature of the object after 3 hours. Round your answer to the nearest degree.

Solution: Newton's Law of Cooling is

$$\frac{dT}{dt} = k(T - S)$$

where T is the temperature of the object, t is time, and S is the ambient temperature (of the room in this case). Hence, our differential equation is

$$\frac{dT}{dt} = k(T - 75).$$

We find the general solution to this equation.

We separate our variables and integrate:

$$\begin{aligned}
\frac{dT}{dt} &= k(T - 75) \\
\Rightarrow \quad \frac{1}{T-75}\,dT &= k\,dt \\
\Rightarrow \quad \int \frac{1}{T-75}\,dT &= \int k\,dt \\
\Rightarrow \quad \ln(T - 75) &= kt + C \\
\Rightarrow \quad T - 75 &= e^{kt+C} = Ce^{kt} \\
\Rightarrow \quad T &= Ce^{kt} + 75
\end{aligned}$$

Now, since we know

$$T = Ce^{kt} + 75,$$

we use $T(0) = 1000$ and $T(1) = 500$ to solve for C and k. Write

$$\underbrace{1000}_{T(0)} = Ce^{k\cdot 0} + 75$$
$$\Rightarrow \quad 925 = C\underbrace{e^0}_{1}$$

Hence,

$$T = 925e^{kt} + 75.$$

Next,

$$\underbrace{500}_{T(1)} = 925e^{k\cdot 1} + 75$$
$$\Rightarrow \quad 425 = 925e^k$$
$$\Rightarrow \quad \frac{425}{925} = e^k$$
$$\Rightarrow \quad \frac{17}{37} = e^k$$
$$\Rightarrow \quad \ln\left(\frac{17}{37}\right) = k$$

Therefore,

$$T = 925e^{\ln(17/37)t} + 75.$$

Finally, we compute $T(3)$,

$$T(3) = 925e^{\ln(17/37)\cdot 3} + 75 \approx \boxed{165°\text{F}}.$$